An Actuarial Approach to Financial Risk Measures and Insurance Pricing

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Abstract

Starting from the continuity condition defined for a Bernoulli random variable X_{qa} ($prob(X_{qa} = a) = q, prob(X_{qa} = 0) = 1 - q$) such that the risk measure $\kappa(X_{qa})$ is strictly increasing in q for fixed a, and such that $\kappa(X_{0a}) = 0, \kappa(X_{1a}) = a$ for $0 < q < 1$, then the risk measure κ is iterative if and only if $v(\kappa(X)) = E[v(X)]$ which is known as a mean value principle. In case v is an exponential function, the optimal situation (minimal risk) for the gain $E[\phi(X)X] - X$ gives rise to an Esscher measure $E[Xe^{\alpha X}]/E[e^{\alpha X}]$.

Given the reserve t for a particular insurance portfolio, the random variable describing the dangerousness of the portfolio, in case of an additional solvency buffer $\rho - t$, is given by $\frac{(X-t)_+}{\rho(X)-t}$. Use of the mean value principle as a risk measure for the quantification of the tail risk allows us to define a set of risk measures that are comparable $\rho_1(X) \leq \rho_2(X)$. Restricting the set of admissible measures to those who result in upper bounds for the VaR, a special attractive set of measures is obtained. A pseudo Esscher transform respects the subadditivity property is obtained. The properties of the resulting risk measures are analyzed. In case the reserve is not fixed in advance, the smallest risk measures are derived and their properties are analyzed.